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Signal Processing Model for Radiation Transport

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1 Introduction

This note describes the design of a simplified gamma ray transport model for use in designing a sequential Bayesian signal processor for low-count detection and classification. It uses a simple one-dimensional geometry to describe the emitting source, shield effects, and detector (see Fig. 1). At present, only Compton scattering and photoelectric absorption are implemented for the shield and the detector. Other effects may be incorporated in the future by revising the expressions for the probabilities of escape and absorption. Pair production would require a redesign of the simulator to incorporate photon correlation effects. The initial design incorporates the physical effects that were present in the previous event mode sequence simulator created by Alan Meyer. The main difference is that this simulator transports the rate distributions instead of single photons. Event mode sequences and other time-dependent photon flux sequences are assumed to be marked Poisson processes that are entirely described by their rate distributions. Individual realizations can be constructed from the rate distribution using a random Poisson point sequence generator.

2 Multiple scattering model for shield and detector

The mathematical models used for photon transport in the shield and detector are identical. The only difference is the quantity of interest for the output. For the shield we require the rate distribution of the photons that escape, while for the detector we require the rate distribution of the deposited energy from both the absorbed and scattered photons. Thus we will first derive a general model for transport within a material that incorporates the probability that a photon will escape the material, the probability that it scatters into a different energy, and the probability it is absorbed and produces a photoelectron. These probabilities are

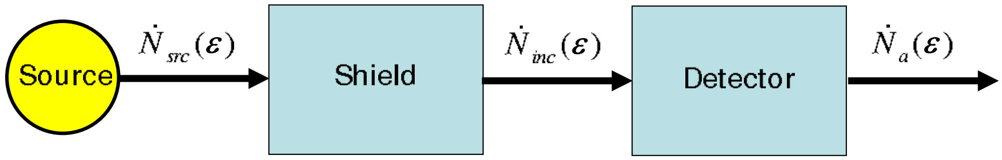

Figure 1: Block diagram of simulation for a single shield and detector. The source emission is characterized a rate distribution over energy: $\dot{N}_{src}(\varepsilon)$ photons per MeV per second. The shield converts this to a rate distribution of $\dot{N}_{inc}(\varepsilon)$ incident on the detector. The output of the detector is the rate distribution of photoelectrons $\dot{N}_a(\varepsilon)$.

characteristic of both the geometry and the material composition. The model is intended to describe a fixed source-shield-detector geometry and is essentially one-dimensional. Changes in source, shield, or detector positions require changing the input probabilities. It also assumes the photons are uncorrelated, so it cannot incorporate pair production.

The basic model begins with the rate distribution $\dot{N}_{inc}(\varepsilon)$ of photons incident on the shield or detector material. This is defined such that the expected number of photons with energies in the interval $(\varepsilon, \varepsilon + d\varepsilon)$ arriving in the time interval $(t, t + dt)$ is $\dot{N}_{inc}(\varepsilon)\, d\varepsilon\, dt$. A fraction $f_a(\varepsilon)$ of the photons in the interval $(\varepsilon, \varepsilon + d\varepsilon)$ will be absorbed, producing photoelectrons with a rate distribution of $\dot{N}_1^{(a)}(\varepsilon) = f_a(\varepsilon)\dot{N}_{inc}(\varepsilon)$. The remaining photons $\overline{f_a}(\varepsilon)\dot{N}_{inc}(\varepsilon)$ either escape the material or scatter.[1] We can write the number that escape as $\dot{N}_1^{(e)}(\varepsilon) = f_e(\varepsilon)\overline{f_a}(\varepsilon)\dot{N}_{inc}(\varepsilon)$, where $f_e(\varepsilon)$ is the fraction that escape. This leaves $\overline{f_a}(\varepsilon)\overline{f_e}(\varepsilon)\dot{N}_{inc}(\varepsilon)$ photons that undergo scattering to different energies.

Let $f_s(\varepsilon|\varepsilon')$ represent the fraction of photons at energy ε' that are scattered to energy ε by one interaction with the material. For Compton scattering the energy of the scattered photon is always less than the incident photon so that $f_s(\varepsilon|\varepsilon') = 0$ for $\varepsilon \geq \varepsilon'$. Using this we can write the rate distribution of singly scattered photons as

$$\dot{N}_1^{(s)}(\varepsilon) = \int_{\varepsilon}^{\infty} f_s(\varepsilon|\varepsilon')\overline{f_a}(\varepsilon')\overline{f_e}(\varepsilon')\dot{N}_{inc}(\varepsilon')\, d\varepsilon' \; . \tag{1}$$

For each photon scattered from energy ε' to energy ε, an electron with energy $\varepsilon' - \varepsilon$ is produced. We can then write the rate distribution of Compton scattered electrons as

$$\dot{N}_1^{(s)}(\varepsilon) = \int_{\varepsilon}^{\infty} f_c(\varepsilon|\varepsilon')\overline{f_a}(\varepsilon')\overline{f_e}(\varepsilon')\dot{N}_{inc}(\varepsilon')\, d\varepsilon' \; , \tag{2}$$

where $f_c(\varepsilon|\varepsilon') = f_s(\varepsilon' - \varepsilon|\varepsilon')$. For convenience let $K(\varepsilon, \varepsilon') = f_s(\varepsilon|\varepsilon')\overline{f_a}(\varepsilon')\overline{f_e}(\varepsilon')$ and define the integral operator $\mathbf{K}$ as

$$\mathbf{K}g(\varepsilon) = \int_{\varepsilon}^{\infty} K(\varepsilon, \varepsilon')g(\varepsilon')\, d\varepsilon' \; . \tag{3}$$

Then we can write $\dot{N}_1^{(s)}(\varepsilon) = \mathbf{K}\dot{N}_{inc}(\varepsilon)$.

The singly scattered photons can escape, be absorbed, or scattered again, so we can write

$$\dot{N}_2^{(a)}(\varepsilon) = f_a(\varepsilon)\dot{N}_1^{(s)}(\varepsilon) \; , \tag{4}$$

$$\dot{N}_2^{(e)}(\varepsilon) = f_e(\varepsilon)\overline{f_a}(\varepsilon)\dot{N}_1^{(s)}(\varepsilon) \; , \tag{5}$$

$$\dot{N}_2^{(c)}(\varepsilon) = \int_{\varepsilon}^{\infty} f_c(\varepsilon|\varepsilon')\overline{f_a}(\varepsilon')\overline{f_e}(\varepsilon')\dot{N}_1^{(s)}(\varepsilon')\, d\varepsilon' \; , \tag{6}$$

$$\dot{N}_2^{(s)}(\varepsilon) = \mathbf{K}\dot{N}_1^{(s)}(\varepsilon) = \mathbf{K}^2\dot{N}_{inc}(\varepsilon) \; . \tag{7}$$

For the n^{th} order scattering we have

$$\dot{N}_n^{(a)}(\varepsilon) = f_a(\varepsilon)\dot{N}_{n-1}^{(s)}(\varepsilon) \; , \tag{8}$$

$$\dot{N}_n^{(e)}(\varepsilon) = f_e(\varepsilon)\overline{f_a}(\varepsilon)\dot{N}_{n-1}^{(s)}(\varepsilon) \; , \tag{9}$$

$$\dot{N}_n^{(c)}(\varepsilon) = \int_{\varepsilon}^{\infty} f_c(\varepsilon|\varepsilon')\overline{f_a}(\varepsilon')\overline{f_e}(\varepsilon')\dot{N}_{n-1}^{(s)}(\varepsilon')\, d\varepsilon' \; , \tag{10}$$

$$\dot{N}_n^{(s)}(\varepsilon) = \mathbf{K}\dot{N}_{n-1}^{(s)}(\varepsilon) = \mathbf{K}^n\dot{N}_{inc}(\varepsilon) \; . \tag{11}$$

[1] Notation: A bar over a fraction indicates the complement, $\overline{f_a}(\varepsilon) = 1 - f_a(\varepsilon)$

To obtain the total rate distributions of escaped and absorbed photons, we sum over all orders:

$$\dot{N}^{(a)}(\varepsilon) = \sum_{n=1}^{\infty} \dot{N}_n^{(a)}(\varepsilon) = f_a(\varepsilon) \left[\dot{N}_{inc}(\varepsilon) + \sum_{n=1}^{\infty} \dot{N}_n^{(s)}(\varepsilon) \right] , \tag{12}$$

$$\dot{N}^{(e)}(\varepsilon) = \sum_{n=1}^{\infty} \dot{N}_n^{(e)}(\varepsilon) = f_e(\varepsilon)\overline{f_a}(\varepsilon) \left[\dot{N}_{inc}(\varepsilon) + \sum_{n=1}^{\infty} \dot{N}_n^{(s)}(\varepsilon) \right] , \tag{13}$$

$$\dot{N}^{(c)}(\varepsilon) = \sum_{n=1}^{\infty} \dot{N}_n^{(c)}(\varepsilon) = \int_{\varepsilon}^{\infty} f_c(\varepsilon|\varepsilon')\overline{f_a}(\varepsilon')\overline{f_e}(\varepsilon') \left[\dot{N}_{inc}(\varepsilon) + \sum_{n=1}^{\infty} \dot{N}_n^{(s)}(\varepsilon) \right] d\varepsilon' . \tag{14}$$

The total rate distributions depend on the sum over all orders of scattering. Define

$$S(\varepsilon) = \dot{N}_{inc}(\varepsilon) + \sum_{n=1}^{\infty} \dot{N}_n^{(s)}(\varepsilon) = \left(\sum_{n=0}^{\infty} \mathbf{K}^n \right) \dot{N}_{inc}(\varepsilon) , \tag{15}$$

then we have

$$\dot{N}^{(a)}(\varepsilon) = f_a(\varepsilon)S(\varepsilon) , \tag{16}$$

$$\dot{N}^{(e)}(\varepsilon) = f_e(\varepsilon)\overline{f_a}(\varepsilon)S(\varepsilon) , \tag{17}$$

$$\dot{N}^{(c)}(\varepsilon) = \int_{\varepsilon}^{\infty} f_c(\varepsilon|\varepsilon')\overline{f_a}(\varepsilon')\overline{f_e}(\varepsilon')S(\varepsilon')\, d\varepsilon' . \tag{18}$$

The sum $S(\varepsilon)$ is the Neumann series solution of the operator equation $S = \dot{N}_{inc} + \mathbf{K}S$, or expanding the operator

$$S(\varepsilon) = \dot{N}_{inc}(\varepsilon) + \int_{\varepsilon}^{\infty} K(\varepsilon, \varepsilon')S(\varepsilon')\, d\varepsilon' . \tag{19}$$

By solving this equation directly, we can calculate the rate distributions of both escaped photons and absorbed photons (photoelectrons) to all orders of scattering. In application, we discretize the energy values ε based on the energy resolution of the detector. The rate distributions become vectors, and the kernel $K(\varepsilon, \varepsilon')$ becomes a matrix. The integral equation (19) becomes a matrix equation that can be solved numerically. In summary, we can model photon transport through material (shield or detector) to all orders of scattering by solving the integral equation (19) for $S(\varepsilon)$ with the rate distribution of the incident photons $\dot{N}_{inc}(\varepsilon)$ as input. The rate distribution of the escaped photons (shield) are then given by $\dot{N}^{(e)} = f_e\overline{f_a}S$. The rate distrition of absorbed photons is $\dot{N}^{(a)} = f_aS$. Figure 2 is a diagrammatic representation of the model. However, the rate of deposited energy for the detector requires more calculation.

Though we have shown how to calculate the rate distribution of photoelectrons and Compton scattered electrons, these are only indirectly related to the detector response to deposited energy. A typical scenario for a photon incident on the detector would be to Compton scatter two times, each time shifting its energy downward, then be absorbed by an atom. This would produce two Compton electrons and one photoelectron, whose combined energy is equal to the incident photon energy. Though the electrons are produced at different times, the difference is too small for a detector to distinguish. Instead, the detector measures a single impulse with energy equal to the total energy of the incident photon. Another typical scenario would be three Compton scatters, then the photon escapes the detector. In this case, the detector measures a single impulse with energy equal to the combined energies of the Compton electrons, which is smaller than the incident photon. The energy carried by the escaped photon is not measured. Thus the detected energy depends on the scattering history of the incident photon. If the photon does not escape the detector, then all its energy is deposited in the detector regardless of how many times it scatters before being absorbed. If the photon escapes, then the detector measures only the difference between incident and escaped energy. Thus $\dot{N}^{(e)}(\varepsilon)$ and $\dot{N}^{(a)}(\varepsilon)$ are required inputs for calculating the rate distribution of the detector $\dot{N}^{(d)}(\varepsilon)$, but we must also take into account photon history.

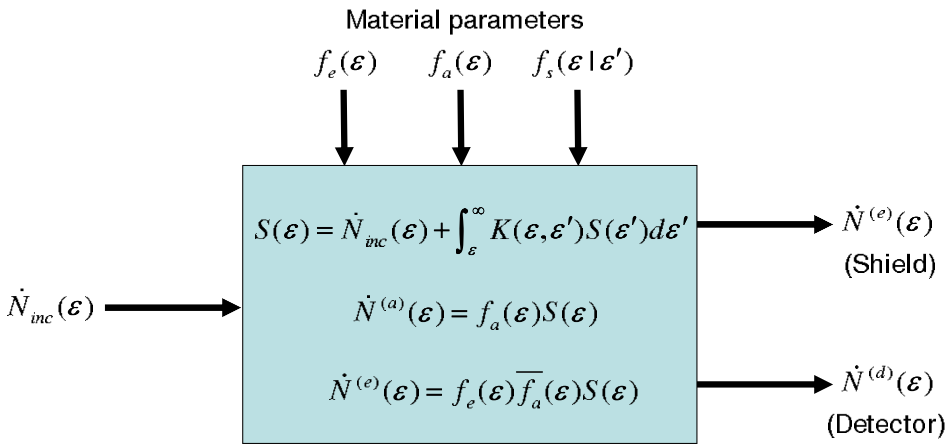

Figure 2: Diagram of photon transport model for a shield or detector. The model calculates either the rate distribution of photons that escape the material $\dot{N}^{(e)}(\varepsilon)$ (shield) or the rate distribution of the absorbed energy $\dot{N}^{(d)}(\varepsilon)$ (detector) given the rate distribution of incident photons $\dot{N}_{inc}(\varepsilon)$.

Consider a monoenergetic incident photon distribution of unit amplitude $\dot{N}_{inc}(\varepsilon|\varepsilon_0) = \delta(\varepsilon - \varepsilon_0)$. The rate distribution of absorbed photons (photoelectrons) and escaped photons is given by

$$\dot{N}^{(a)}(\varepsilon|\varepsilon_0) = f_a(\varepsilon) S(\varepsilon|\varepsilon_0)\,, \tag{20}$$

$$\dot{N}^{(e)}(\varepsilon|\varepsilon_0) = f_e(\varepsilon)\overline{f_a}(\varepsilon) S(\varepsilon|\varepsilon_0)\,, \tag{21}$$

where

$$S(\varepsilon|\varepsilon_0) = \dot{N}_{inc}(\varepsilon|\varepsilon_0) + \int_{\varepsilon}^{\varepsilon_0} K(\varepsilon,\varepsilon') S(\varepsilon'|\varepsilon_0\, d\varepsilon'\,. \tag{22}$$

The detector response is a combination of two terms. The first term represents the photons that are absorbed. Every photon that contributes to $\dot{N}^{(a)}(\varepsilon|\varepsilon_0)$ entered with energy ε_0. These photons have lost all their energy to the detector and produce a *photopeak* at energy ε_0:

$$\dot{N}^{(d)}(\varepsilon|\varepsilon_0) = \int_0^{\varepsilon_0} \dot{N}^{(a)}(\varepsilon'|\varepsilon_0)\, d\varepsilon'\, \delta(\varepsilon - \varepsilon_0) \,+\, \text{second term}\,. \tag{23}$$

The second term comes from the scattered photons that escape from the detector. For every photon of energy ε' that escapes from the detector, an energy of $\varepsilon_0 - \varepsilon'$ has been absorbed by the detector. Thus the rate distribution of energy absorption due to photons scattered out of the detector is $\dot{N}^{(e)}(\varepsilon_0 - \varepsilon|\varepsilon_0)$. Thus the rate distribution for the detector response is

$$\dot{N}^{(d)}(\varepsilon|\varepsilon_0) = \int_0^{\varepsilon_0} \dot{N}^{(a)}(\varepsilon'|\varepsilon_0)\, d\varepsilon'\, \delta(\varepsilon - \varepsilon_0) + \dot{N}^{(e)}(\varepsilon_0 - \varepsilon|\varepsilon_0)\,. \tag{24}$$

Note that photons that escape the detector without scattering ($\dot{N}^{(e)}(\varepsilon_0|\varepsilon_0)$) contribute zero energy to the detector and can be ignored. For a more general rate distribution of incident photons we use the linearity of the solution for the monoenergetic, unit amplitude case to write

$$\dot{N}^{(d)}(\varepsilon) = \int_0^{\infty} \dot{N}_{inc}(\varepsilon')\dot{N}^{(d)}(\varepsilon|\varepsilon')\, d\varepsilon'\,. \tag{25}$$

Though one can substitute the expression for $\dot{N}^{(d)}(\varepsilon|\varepsilon')$ into the above integral, it still leads to integrals over $\dot{N}^{(a)}(\varepsilon|\varepsilon')$ and $\dot{N}^{(e)}(\varepsilon|\varepsilon')$. Thus we still must solve (22) for each value of ε_0 in the support of $\dot{N}_{inc}$, then superpose the solutions using (25).

3 Compton scattering and probability of escape

Two of the inputs to the transport model are the probability of escape $f_e(\varepsilon)$ and the conditional scattering probability $f_s(\varepsilon|\varepsilon')$. We will develop both of these based on Compton scattering as the main mechanism of interaction of the photons with the material. We model the material as a cloud of electrons with average density of N_e electrons per unit volume. Let $\sigma(\varepsilon)$ be the total Compton cross-section for incident photons of energy ε. The linear scattering coefficient $\mu(\varepsilon)$ for a homogeneous material of mass density ρ and effective atomic number Z is[2]

$$\mu(\varepsilon) = \rho N_A \frac{Z\sigma(\varepsilon)}{M} \; , \tag{26}$$

where N_A is Avogadro's number and M is the molar mass of the material. The probability density function of photon path lengths R in the material is exponential[2]:

$$p(R|\varepsilon) = \frac{1}{\lambda(\varepsilon)} e^{-R/\lambda(\varepsilon)} \; , \tag{27}$$

where $\lambda(\varepsilon) = 1/\mu(\varepsilon)$ is the mean-free-path (MFP) for photons of energy ε. If L is the characteristic size of the material, then all photons with path length $R > L$ will escape. Thus we estimate the fraction of photons that escape as

$$f_e(\varepsilon) = \int_L^\infty p(R)\, dR = e^{-L/\lambda(\varepsilon)} \; . \tag{28}$$

This neglects shape effects and would not be strictly valid for photons that originate within the materials, scattered from other energies. However, we can start with the exponential form and modify the characteristic size L_{eff} of the material to provide a good approximation for a particular configuration:

$$f_e(\varepsilon) = e^{-L_{eff}/\lambda(\varepsilon)} \; . \tag{29}$$

We will calculate the conditional scattering probability using the differential cross-section for Compton scattering and the relationship between output energy and scattering angle. For a photon of energy ε' scattering from an electron, the output energy ε is given by

$$\varepsilon = \frac{\varepsilon'}{1 + \frac{\varepsilon'}{\varepsilon_r}(1 - \cos\theta)} \; , \tag{30}$$

where $\epsilon_r = m_0 c^2$, and θ is the angle between the direction of the incident photon and direction of the scattered photon. The differential scattering cross-section for unpolarized incident photons is given by the Klein-Nishina formula:

$$d\sigma_s = \frac{r_0^2}{2} d\Omega(\theta) \frac{1 + \cos^2\theta}{[1 + \alpha(1 - \cos\theta)]^2} \left\{ 1 + \frac{\alpha^2 (1 - \cos\theta)^2}{(1 + \cos^2\theta)\,[1 + \alpha(1 - \cos\theta)]} \right\} \; , \tag{31}$$

where $\alpha = \varepsilon'/\varepsilon_r$, and $d\Omega(\theta)$ is the differential solid angle around θ. The differential cross-section is the ratio of power $d\dot{E}(\theta)$ scattered into the solid angle $d\Omega$ around θ to the incident photon intensity I_0. The scattered power is related to the rate of photons scattered at angle θ: $d\dot{E}(\theta) = \varepsilon\, d\dot{N}(\theta)$. The intensity is proportional to the number of incident photons per second crossing a circle of radius r_0: $I_0 = \dot{N}\varepsilon/\pi r_0^2$. Thus we can write

$$\frac{d\dot{N}}{\dot{N}} = \frac{d\Omega}{2\pi} \frac{1 + \cos^2\theta}{1 + \alpha(1 - \cos\theta)} \left\{ 1 + \frac{\alpha^2 (1 - \cos\theta)^2}{(1 + \cos^2\theta)\,[1 + \alpha(1 - \cos\theta)]} \right\} \tag{32}$$

where we have used equation (30) to simplify the ratio ε'/ε. We can express θ in terms of the incident and scattered energies using (30):

$$\cos\theta = 1 - \varepsilon_r \left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon'} \right) \; . \tag{33}$$

The differential solid angle $d\Omega(\theta)$ can be written

$$d\Omega(\theta) = 2\pi \sin\theta\, d\theta = -2\pi\, d(\cos\theta) = -2\pi \frac{\varepsilon_r d\varepsilon}{\varepsilon^2} \,. \tag{34}$$

The conditional scattering probability specifies the fraction of photons scattered from an incident energy ε' to the energy ε, *i.e.* $d\dot{N}/\dot{N} = f_s(\varepsilon|\varepsilon')\, d\varepsilon' = f_s(\varepsilon|\varepsilon')(d\varepsilon'/d\varepsilon)\, d\varepsilon$. Combining this together along with $d\varepsilon/d\varepsilon' = (\varepsilon/\varepsilon')^2$ gives

$$f_s(\varepsilon|\varepsilon') = C\frac{\varepsilon_r \varepsilon}{\varepsilon'^3}\left[2 - 2\varepsilon_r\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon'}\right) + (\varepsilon_r^2 + \varepsilon\varepsilon')\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon'}\right)^2\right] , \tag{35}$$

where C is a normalization constant. Note from equation (30) the scattered energy ε satisfies the bounds

$$\varepsilon_0 = \frac{\varepsilon'}{1 + 2\varepsilon'/\varepsilon_r} \le \varepsilon \le \varepsilon' \,, \tag{36}$$

so the normalization constant can be obtained by requiring that the integral of $f_s(\varepsilon|\varepsilon')$ in this interval to be unity. This requirement gives

$$C = \frac{3\varepsilon'}{2\varepsilon_r}\left(1 + \frac{2\varepsilon'}{\varepsilon_r}\right)^3\left[-3 - \frac{15\varepsilon'}{\varepsilon_r} - \frac{18\varepsilon'^2}{\varepsilon_r^2} + \frac{6\varepsilon'^3}{\varepsilon_r^3} + \frac{16\varepsilon'^4}{\varepsilon_r^4} + \frac{3\varepsilon_r}{2\varepsilon'}\left(1 + \frac{2\varepsilon'}{\varepsilon_r}\right)^3 \ln\left(1 + \frac{2\varepsilon'}{\varepsilon_r}\right)\right]^{-1} , \tag{37}$$

which completes the specification of the conditional scattering probability.

4 Absorption model

Photons are absorbed through the photoelectric effect. Evans[1] notes that a crude approximation of the cross-section is Z^4/ε^3 (Z is the atomic number for the material). The simplest expression for $f_a(\varepsilon)$ that obeys the condition $f_a(\varepsilon) \to 1$ as $\varepsilon \to 0$ would be

$$f_a(\varepsilon) = \frac{1}{1 + (\varepsilon/\varepsilon_a)^3} \,, \tag{38}$$

where ε_a sets the transition energy between full absorption (small ε) and low absorption ($f_a \sim 1/\varepsilon^3$). More detailed models certainly exist but measured cross-sections might be more practical for actual applications.

5 Generation of simulated event mode sequences

Simulated event mode sequences (EMS) for the detector can be generated from the final rate distribution $\dot{N}_a(\varepsilon)$ using a Poisson point realization generator. From the definition of the rate distribution the probability of n events with energy ε in the time interval $(t_0, t_0 + t)$ is given by

$$P\{N(t_0, t_0 + t|\varepsilon) = n\} = \frac{(\dot{N}(\varepsilon)t)^n}{n!}\, e^{-\dot{N}(\varepsilon)t} \,. \tag{39}$$

Equivalently, the probability distribution of interarrival times at energy ε is

$$f_{\Delta t}(\Delta t|\varepsilon) = \dot{N}(\varepsilon)e^{-\dot{N}(\varepsilon)\Delta t} \,. \tag{40}$$

To construct a realization of an event mode sequence we first discretize the energy domain into a finite set of bins $\varepsilon \Rightarrow \varepsilon_j : j = 1, 2, \ldots, J$. For energy ε_j generate a set of Poisson points $t_m^{(j)} : m = 1, 2, \ldots, M_j$ whose

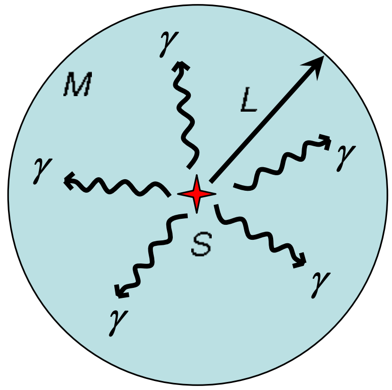

Figure 3: Configuration of simple example: a point source $\boldsymbol{S}$ emitting 900 keV photons (γ) is placed at the center of a sphere $\boldsymbol{M}$ of hydrogen with radius $\boldsymbol{L}$
.

interarrival times are distributed according to equation (40). Assign the weight ε_j to these points. Finally sum over the energy bins to obtain a realization of the EMS:

$$\mathrm{EMS} = \sum_{j=1}^{J} \varepsilon_j \sum_{m=1}^{M_j} \delta\left(t - t_m^{(j)}\right) \ . \tag{41}$$

With this procedure we can generate any number of realizations from the rate distribution $\dot{N}_a(\varepsilon)$ calculated from the transport model.

6 Simple example

To illustrate the model, consider the simple scenario in figure 3. A point source of 900 MeV photons (gamma rays) is enclosed in a sphere of hydrogen. The sphere radius L is set to the photon mean-free-path λ for Compton scattering. We calculate the rate distribution of escaped photons $\dot{N}^{(e)}(\varepsilon)$ using the simple model for the shield. This is compared to the photon flux distribution calculated from COG, a Monte-Carlo radiation transport code. Absorption is negligible at these energies so we can set $f_a = 0$ in the model. Figure 4 shows the comparison for three values of effective size, $L_{eff} = L = \lambda$, $1.43L$, $2.12L$. These correspond to $f_e(\varepsilon) = 1/e$, 0.24, , 0.12, constant over all energies (eq. 29). The value of $\dot{N}^{(e)}(\varepsilon)$ at the photopeak (900 keV) was normalized to the COG result for each case. The best result is obtained for $L_{eff} = 1.43L$ ($f_e = 0.24$). No attempt was made to optimize the value of L_{eff}. The agreement is quite good, with the greatest variation occuring at the low energies. The small slope discontinuity in the COG result at the Compton backsdcatter energy of 200 keV is reproduced by the model.

Acknowledgment

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[1] Robley D. Evans, *The Atomic Nucleus*, Krieger Publishing Company, Malabar, Florida, 1985.

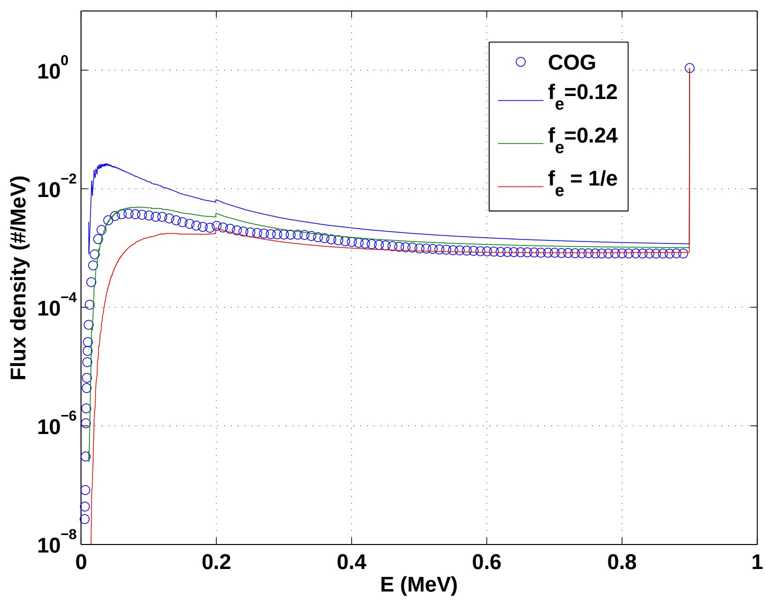

Figure 4: Comparison of rate distribution between COG (circles) and the simple model with $m = +1$ in the Compton scattering model. The values for the escape fraction f_e were chosen to bracket the COG results. $f_e = 1/e$ is the nominal value based on the radius of the hydrogen sphere in COG.

[2] Ivan Lux and Laxzlo Koblinger, *Monte Carlo Particle Transport Methods: Neutron and Photon Calculations*, CRC Press, Boca Raton, Florida, 1991.